

## A Method of Improving the Response of Waveguide Directional Couplers\*

This communication describes a technique for decreasing the frequency variation of coupling of multi-hole broadwall waveguide directional couplers. The usual curve of coupling vs frequency is shown in Fig. 1<sup>1</sup> and has a peak-to-peak variation of 1.0 db. The proposed method, which is similar to the use of multiple sections in coaxial couplers,<sup>2</sup> perturbs the coupled voltage by adding a small voltage that is in phase at midband where the coupling is looser and out of phase near the band edges. The required perturbation is produced by means of the coupling structure shown in Fig. 2, where the phase of one coupled voltage is delayed with respect to the other coupled voltage because of an added path length  $\Delta L$ . The coupled power is proportional to:

$$P_{c\alpha} \left[ 1 + \left( \frac{k_2}{k_1} \right)^2 + 2 \frac{k_2}{k_1} \cos \phi \right] \quad (1)$$

where

$$\phi = \frac{2\pi\Delta L}{\lambda_g}. \quad (2)$$

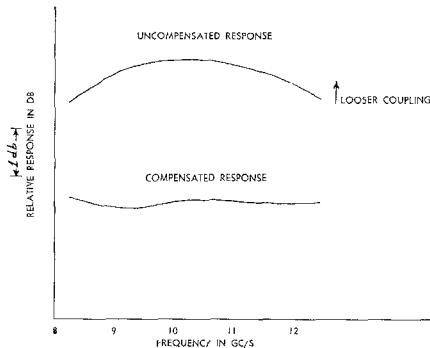


Fig. 1—Compensated and uncompensated coupler response.

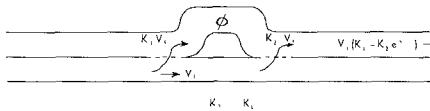


Fig. 2—Proposed method of compensation.

The above equation assumes that the compensating array has the same frequency dependence as does the main array.

The computed compensated response is also shown in Fig. 1 for  $\Phi=360$  at 10.2 GHz and  $k_2/k_1=0.06$ . The original deviation of 0.9 db has been reduced to less than 0.2 db.

One should note that the directivity of the compensating array need not be as great as that of the main array because of its relative decoupling.

LEONARD SWEET  
General Microwave Corporation  
Farmingdale, N. Y.

## The Characteristic Impedance of Square Coaxial Line\*

### INTRODUCTION

Recent attempts to design a precision coaxial to strip line transformer have raised the need to know accurately the characteristic impedance of square coaxial line (*i.e.*, a concentric line, as shown in Fig. 1, having inner and outer conductors of square cross section) which forms an intermediate part of the proposed transformer.

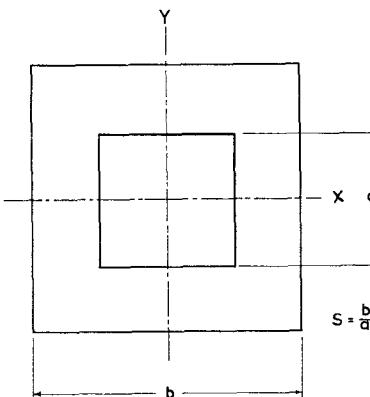


Fig. 1—Cross section of square coaxial line.

Solutions to this problem exist in the literature ranging from the purely empirical [1] through the approximate analytic [2] to the exact [3]. For precise work the empirical and approximate analytic solutions are not generally sufficiently accurate while the exact solution, involving elliptic integrals, is computationally tedious and does not give a direct answer to the usual question of what dimensions must be used to obtain some specified characteristic impedance.

### NUMERICAL SOLUTION

Recently as part of a more general task attention has been given to the programming of an IBM 7090 computer to solve boundary value problems in Laplace's equation by the square net method [4]. This effectively reduces the problem of establishing the potentials at the nodal points of the mesh to that of solving a group of simultaneous linear finite difference equations—there being as many equations as meshes in the net. The method of solution is based on the usual matrix techniques but makes use of the special band nature of the matrix to speed inversion. With the present program it is possible to cope with up to about 1000 mesh problems in realistic computing times.

Obviously if the potentials in the space between a pair of conducting surfaces can be established it is a simple matter to extend the work to obtain the charge on the surfaces and hence the capacity. Since capacity per unit length is simply related to the characteristic impedance of the corresponding transmission line by

$$Z_0 = \frac{1}{2C} \quad (1)$$

\* Received August 5, 1963.

<sup>1</sup> L. I. Kent, "The Optimum Design of Multielement Directional Couplers," Master's thesis, Polytechnic Institute of Brooklyn, N. Y., June, 1954.

<sup>2</sup> E. F. Barnett, P. D. Lacy, and B. M. Oliver, "Principles of Directional Coupling in Reciprocal Systems," presented at the Symposium on Modern Advances in Microwave Techniques, Polytechnic Institute of Brooklyn, N. Y.; November 8-10, 1954.

where

$$\begin{aligned} Z_0 &= \text{the characteristic impedance in ohms} \\ C &= \text{the capacity per unit length (fd/m)} \\ v &= \text{the velocity of light (2.9979} \times 10^8 \text{ m/sec)} \end{aligned}$$

this method has an obvious application in the solution of the square coaxial line problem.

This has been done for evacuated square coaxial lines of the side length ratios (ratio of outer to inner conductor side length) shown in Table I. An effective increase in the fineness of mesh subdivision for a given program capacity can be had by making use of obvious symmetry and treating only a quadrisection. To further improve the accuracy each problem was worked out three times on an increasingly fine net and the three solutions subjected to Richardson extrapolation [5] to obtain the figure given in the tables.

### ACCURACY

It is of interest to attempt to estimate the accuracy of this data. It follows from Thomson's theorem [6] that the values of capacity derived in this way must always be too high and hence the characteristic impedances too low. The only way to say by how much would appear to be to compare these solutions with what little numerical data is available from the various analytic attacks.

Chen [2] develops a conformal transformation solution for the capacity of an isolated square corner between two infinite right angle plates and then makes the assumption that a square coaxial line may be considered to be made up of eight paralleled capacities, four parallel plate and four isolated corner. This should give good results for low side length ratios where the corners are well spaced and at the same time make only a small contribution to the total capacity. For a line with  $s=1.25$  Chen's results give  $Z_0=11.005$  ohms; comparison with Table I indicates agreement to around 0.1 per cent.

Anderson [3] gives an exact conformal transformation solution and uses it to calculate to four figures one numerical example with  $s=3.0$ . Interpreted as a characteristic impedance this solution gives  $Z_0=60.75$  ohms. Comparison with Table I shows a close concurrence within 0.4 per cent. Skiles and Higgins [7] have also attacked this problem by a type of variational approach and also give a numerical result for  $s=3.0$ , obtaining somewhat inconsistently with Anderson upper and lower bounds of 60.65 and 60.47 ohms, respectively. The value shown in Table I will be seen to be contained between these limits and to agree with their mean within around 0.05 per cent.

From this evidence it seems fair and perhaps even conservative to claim an average accuracy within  $\frac{1}{2}$  per cent for the figures given in Table I.

### APPLICATION TO DESIGN

To make this information more useful for design purposes the basic data given in Table I has been subjected to inverse (Lagrangian) interpolation to prepare a table of side length ratios corresponding to specified characteristic impedances, advancing in one ohm steps from 0 to 100 ohms.

TABLE I

Side Length Ratio	Characteristic Impedance	Side Length Ratio	Characteristic Impedance
5:4	10.993	7:2	69.687
3:2	20.637	4:1	77.630
5:3	26.441	9:2	84.704
2:1	36.771	5:1	90.981
5:2	49.754	11:2	96.718
3:1	60.528	6:1	101.852

TABLE II  
TABULATION OF CHARACTERISTIC IMPEDANCE OF SQUARE COAXIAL LINE

$Z_0$	$s$										
0.	1.000	17.	1.407	34.	1.907	51.	2.554	68.	3.402	85.	4.522
1.	1.021	18.	1.432	35.	1.940	52.	2.598	69.	3.460	86.	4.599
2.	1.043	19.	1.457	36.	1.974	53.	2.642	70.	3.519	87.	4.676
3.	1.065	20.	1.483	37.	2.008	54.	2.688	71.	3.578	88.	4.755
4.	1.087	21.	1.510	38.	2.043	55.	2.734	72.	3.639	89.	4.836
5.	1.109	22.	1.537	39.	2.078	56.	2.780	73.	3.701	90.	4.918
6.	1.132	23.	1.565	40.	2.114	57.	2.827	74.	3.764	91.	5.002
7.	1.155	24.	1.594	41.	2.151	58.	2.875	75.	3.828	92.	5.086
8.	1.178	25.	1.624	42.	2.188	59.	2.924	76.	3.892	93.	5.172
9.	1.202	26.	1.653	43.	2.226	60.	2.974	77.	3.958	94.	5.259
10.	1.225	27.	1.684	44.	2.265	61.	3.024	78.	4.025	95.	5.347
11.	1.250	28.	1.714	45.	2.304	62.	3.075	79.	4.092	96.	5.436
12.	1.279	29.	1.746	46.	2.344	63.	3.127	80.	4.161	97.	5.525
13.	1.306	30.	1.777	47.	2.384	64.	3.180	81.	4.231	98.	5.616
14.	1.332	31.	1.809	48.	2.426	65.	3.234	82.	4.302	99.	5.709
15.	1.357	32.	1.841	49.	2.468	66.	3.289	83.	4.374	100.	5.805
16.	1.382	33.	1.874	50.	2.511	67.	3.345	84.	4.448	101.	5.907

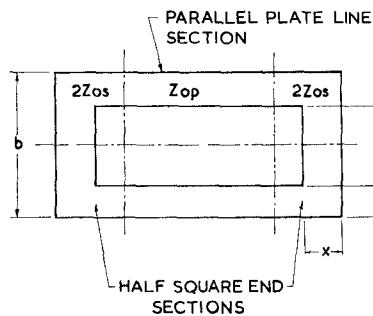


Fig. 2—Development of a square cornered rectangular coaxial line from square section line.

This is presented as Table II. While this introduces the possibility of generating additional errors there is sufficient initial data to suggest that these should remain small. Near the actual data points the basic accuracy of the original work will be preserved. It is pleasing to note the (fortuitous) closeness of the important practical cases of 50, 70 and 75 ohm impedances to basic data points.

When the line is to be filled with a material of dielectric constant  $K_e$  the tables should be entered at an impedance of  $Z_0\sqrt{K_e}$ , where  $Z_0$  is the required characteristic impedance.

The special case of a rectangular line (Fig. 2) having square corner regions is also covered approximately by this work. Such a line may be considered as the paralleled combination of two half square coaxial lines with a sandwich (parallel plate) line. The accuracy of the approximation will depend on how little the introduction of the parallel plate line disturbs the field at the corners and on what proportion of the capacity these ends contribute. No generalization is therefore possible.

Finally for  $s \geq 2.0$  it has been found that the data of Table I is fitted with an error not exceeding 0.5 per cent by the empirical relation

$$Z_0 = 136.7 \log_{10} 0.9259s \quad (2)$$

## ACKNOWLEDGMENT

It is a great pleasure to acknowledge the many helpful discussions with B. P. McDowell of Mathematical Services Group whose expert knowledge of digital programming techniques has played a most significant part in this work. The author also wishes to thank the Chief Scientist of the Australian Department of Supply for permission to publish the work.

HARRY E. GREEN  
Weapons Research Establishment  
Salisbury, South Australia

## REFERENCES

- [1] Y. A. Omar and C. F. Miller, "Characteristic impedance of rectangular coaxial transmission lines," *Trans. AIEE*, vol. 71 (Communication and Electronics, pt. 1), pp. 81-89; January, 1952.
- [2] T.-S. Chen, "Determination of the capacitance, inductance and characteristic impedance of rectangular lines," *IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES*, vol. MTT-8, pp. 510-519; September, 1960.
- [3] G. M. Anderson, "The calculation of the electrical capacitance of coaxial cylinders of rectangular cross-section," *Trans. AIEE*, vol. 69, pt. II, pp. 728-731; 1950.
- [4] A. Thom and C. J. Apelt, "Field Computations in Engineering and Physics," D. Van Nostrand Co., Inc., New York, N. Y., 1st ed., ch. 4, p. 29; 1961.
- [5] R. Culver, "The use of extrapolation techniques with electrical network analogue solutions," *Brit. J. Appl. Phys.*, vol. 3, pp. 376-378; December, 1952.
- [6] R. E. Collin, "Field Theory of Guided Waves," McGraw-Hill Book Co., Inc., New York, N. Y., 1st ed., ch. 1, sec. 1, p. 11; 1960.
- [7] J. J. Skiles and T. J. Higgins, "Determination of the characteristic impedance of U.H.F. coaxial rectangular transmission lines," *Proc. Natl. Electronics Conference*, vol. 10, pp. 97-100, 1954.

Postscript to Two Papers on  
Waffle-Iron Filters\*

## INTRODUCTION

Waffle-iron filters are waveguide "low-pass" filters, that is, they have a wide stop band above their pass band. They are useful for both low-power and high-power applications. A brief history of the development of this filter at Stanford Research Institute is given by Young and Schiffman [1]. A new design principle which makes it possible to extend the pass band from (typically) about 10 per cent to about 40 per cent is also described [1]; the pass band of that filter covers almost the whole of the recommended waveguide band, while its stop band extends from the second to the fourth harmonic, inclusive.

The waffle-iron filter described by Sharp [2] consists of three waffle-iron filters in cascade, to extend the stop band up to the tenth harmonic, inclusive. However, the individual filters had relatively narrow pass bands, and had to be matched to each other by means of quarter-wave transformers.

The purpose of this communication is to show how the wide pass band feature of the one filter [1] was combined with the wide stop band feature of the other filter [2]. At the same time, as the pass band of the original filter [2] was more than doubled, the over-all length of the filter was shortened by almost one half, resulting in an electrically improved as well as a mechanically more compact filter.

The principal design modification made on the original filter [2], was to convert the half-capacitance end sections [2] to half-inductance end sections [1]. This change is simple to make mechanically, but is far-reaching electrically, since the image impedance of the filter now varies with frequency very nearly like the characteristic impedance of the connecting waveguide. This makes it possible to achieve a good match over a very wide pass band. (For a detailed explanation, the reader is referred to Young and Schiffman [1].)

## DESIGN PROCEDURE

The original filter consisted of three separate waffle-iron filters designed for separate and slightly overlapping stop bands. They were cascaded, with quarter-wave transformers placed between them to match their different image impedances. The over-all length of the original filter (shown in Fig. 1 of Sharp [2]) was about 30 inches, which was reduced to about 18 inches in the modified filter.

Two design modifications were undertaken:

- 1) A half section was removed at each end of the three separate filters [2], thus turning the half-capacitance end sections into half-inductance end sections [1], in order to broaden the pass band of each filter.

\* Received June 17, 1963; revised manuscript received August 7, 1963. This work was supported by the Rome Air Development Center under Contract AF 30(602)-2734.